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Structural Optimization—Past, Present, and Future

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Introduction

THE concept of optimization is intrinsically tied to natural phenomena as well as to the human desire to excel. Sir George Cayley (1773-1857) measured the shape of a trout and noted, without mathematical proof, that the trout was ideally proportioned to minimize flow resistance.[1] Theodore von Kármán[2] observed that this is precisely the shape of a low-drag airfoil. Oliver Wendell Holmes (1809-1894), in his classic verse, "The Deacon's Masterpiece; or, The Wonderful One-Hoss Shay," recorded man's desire to produce a uniformly strong, durable product.[3] In this case it was the structural design of a shay to last a hundred years.

Perhaps the first analytical work in structural optimization was by Maxwell in 1869,[4] followed by the better-known work of Michell in 1904.[5] These works provided theoretical lower bounds on the weight of trusses, and, although highly idealized, offer considerable insight into the structural optimization problem and the design process.

The 1940s and early 1950s saw development of component optimization in such works as Shanley's *Weight-Strength Analysis of Aircraft Structures*.[6] Also during this period, availability of the digital computer led to application of linear programming techniques to plastic design of frames, for example, the work of Heyman.[7] This early numerical work is particularly significant in that it used mathematical programming techniques developed in the operations research community to solve structural design problems.

Schmit[8] in 1960 was the first to offer a comprehensive statement of the use of mathematical programming techniques to solve the nonlinear-inequality-constrained problem of designing elastic structures under a multiplicity of loading conditions. This work is significant, not only in that it ushered in an era of structural optimization, but also because it offered a new philosophy of engineering design which is only now beginning to be broadly applied. In Ref. 9 Schmit provides an excellent historical review of the development of this concept.

Although this discussion will emphasize numerical design techniques, it is important to note that there has been an extensive amount of research in analytical methods of design. That work, although sometimes lacking the practicality of being applied to realistic structures, is nonetheless of fundamental importance because it provides insight into the design problem and because it often provides theoretical lower bounds against which more practical designs may be judged. References 10 and 11 provide an extensive review of the state-of-the-art in analytical design techniques.

It is the use of numerical techniques in structural optimization that is emphasized here. The purpose is not to offer a tutorial on optimization or a comprehensive literature survey, although such works are referenced. Rather, it is to look briefly at the short history of modern structural optimization and assess the state-of-the-art from a somewhat more philosophical viewpoint. In this way we may begin to understand the ramifications of this fascinating approach to design. By learning what is now possible and what is not now possible, we may encourage the use of these techniques by practicing designers as well as identify research and development needs of the future.

The Design Problem

The design task is presented here together with basic definitions to provide a common terminology for discussion. The problem is stated in the context of mathematical programming because this provides the most general format for design. Only a brief outline is given here. Reference 12 presents a more general description of mathematical programming techniques as applied to engineering design.

Mathematically, the design task is to find the set of n design variables contained in the vector X that will minimize

$$F(X) \tag{1}$$

subject to:

$$g_j(X) \leq 0 \qquad j=1,m \tag{2}$$

$$h_k(X) = 0 \qquad k=1 \tag{3}$$

$$X_i^l \leq X_i \leq X_i^u \qquad i=1,n \tag{4}$$

The components X_i of X are referred to as "design variables," and the designer is free to change their values to improve the design. In structural optimization the design variables are typically member dimensions, joint coordinates, or the number of plys in a composite laminate.

The function $F(X)$ is called the "objective." While $F(X)$ may be cost or some measure of performance, it is most commonly the weight of the structure.

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Inequality constraints $g_j(X)$ are the response limits imposed on the design. For example, the stress at some point in the structure may be limited to the bound $\bar{\sigma}$ so the constraint function is written in normalized form as

$$(\sigma/\bar{\sigma}) - 1 \leq 0 \tag{5}$$

Other common constraints include limits on deflection, frequency, flutter, and local and system stability as examples. These are one-sided limits that need not be satisfied with precise equality.

Equality constraints $h_k(X)$ are precise requirements that must be met for the design to be acceptable. Equality constraints can include the conditions of equilibrium and compatibility. However, these are usually satisfied as a subproblem, vis-à-vis finite element analysis, rather than within the actual design statement. Consequently, equality constraints are not as common in structural optimization as in other design optimization problems.

The lower and upper bounds, X_i^l and X_i^u, respectively, on the design variables are referred to as "side constraints." These could be included in the general-inequality-constraint set $g(X)$ but are usually treated separately because they directly limit the region of search for the optimum. The most common side constraint is the minimum-gage limit. Note that, although the constraints $g(X)$ and $h(X)$ may not be satisfied at some point in the design process, it is often essential for the side constraints to be satisfied at all times in order to evaluate the other constraints. For example, if the analysis of a structure is attempted with a negative member thickness (violation of the minimum-gage constraint), the calculated response quantities, such as stress and displacement, are not meaningful and indeed may not be computationally obtainable due to matrix singularity in the analysis.

The design statement given here is general, and, at this point, no restrictions have been placed on the nature of the problem. The design variables may be continuous, discrete, or integer. Similarly, the objective and constraint functions may be discontinuous and have discontinuous derivatives. There may be only one unique optimum or there may be many relative optima. How many, and the nature, of the restrictions that must be imposed on the design problem to make its solution tractable will determine the value of any proposed design technique.

The quest for a general, efficient, and reliable method for structural design optimization is the subject here. In the introduction, a historical narrative leading to the age of computers was offered. In the following sections, the concept developments of the 1960s and subsequent refinements of the 1970s are reviewed to provide an understanding of the state-of-the-art. This review leads naturally to an indication of the needs and expectations of the future in structural optimization.

The Magnificent '60's

The setting of the late 1950's was in many ways ideal for major advances in structural design. The space race was well under way, creating a strong demand for lightweight structures and providing the research funds necessary to develop new design techniques. Digital computers were becoming commonly available, and the finite element method was offering the designer a tool for analysis of increasingly complex structures. It is not surprising that Schmit's landmark paper of 1960[8] was the beginning of a remarkable era of research in computer-based design. This work was of particular importance in two respects. First, using a simple example, Schmit demonstrated that the least-weight design may not be fully stressed, where a fully stressed design is one in which each member is stressed to its allowable limit under at least one loading condition. This result was counterintuitive and indeed was a startling departure from the simultaneous-failure-mode approach in common use at the time. Second, through the use of nonlinear programming techniques, he offered a means of obtaining this optimum on the computer. This new method did not require a priori selection of the failure modes, thus offering the designer the freedom to include many possible failure modes, allowing the computer to select those that would influence the design.

This author became involved in structural optimization in the late 1960's and so brings the perspective of one who was not involved in, and indeed was not aware of, the vast research in the field up to that time. The review papers of that period,[13-17] listing references in the hundreds, attest to the dynamic state-of-the-art in this field.

Much of the interest in these new techniques clearly resulted from the fact that the problem statement required little modification for application to structures of general interest. Nonlinear programming techniques of the time usually required that one work with continuous variables and assumed mathematical convexity, but, most important, did not require linearization of what was fundamentally a nonlinear problem. Convexity implies that there are no relative minima, an assumption that had been tacitly accepted before this time anyway. Although for many problems it was desirable to work with discrete variables so that the design could be chosen from available gage sizes, or an integer number of stiffeners could be specified for a cylinder, in most cases an acceptable solution could be achieved by treating the design parameters as continuous variables. Finally, there appeared to be a natural marriage between mathematical programming and finite element methods for linear analysis because gradient (sensitivity) information could be readily obtained. This allowed the use of many of the more powerful gradient-based mathematical programming algorithms. Therefore, the variety and sophistication of design problems that could be pursued appeared inexhaustible.

Mathematical programming techniques were shown to be an effective tool for design of numerous civil, aeronautical, and space structures. Design variables primarily were truss member dimensions, shell thicknesses, and ring and stringer dimensions. These were traditional design variables, but now several could be considered simultaneously. More important, the structure was designed to satisfy multiple and often complex constraints including strength, deflection, stability, frequency, flutter, and postbuckling response limits under a variety of loading conditions.

It is not surprising that many researchers considered these structural synthesis concepts to be a revolutionary change in our approach to design. Because this promising tool was so new, with so much development required to establish the methodology, it was enthusiastically pursued.

By the late 1960's, however, it was becoming apparent that structural synthesis was not being embraced by the professional community, as many people, including this new convert, expected it would be. Some plausible explanations can be offered. First, design is far more complex than analysis, and at that time the finite element method was just becoming generally accepted after approximately fifteen years of development. A new design methodology takes longer to gain general acceptance. Second, structural synthesis represents an integration of engineering and operations research disciplines. Because mathematical programming methods were unknown to the vast majority of engineering researchers, educators, and practitioners, it was unreasonable to expect immediate and widespread acceptance, particularly since nonlinear programming was itself not a mature discipline.

Each explanation was reasonable and required only time and patience to overcome. It was also becoming recognized, however, that there might be a fundamental limit of this new technology. The simplest problem often needed to be analyzed hundreds of times during optimization. If this analysis were time-consuming, as is often the case for large finite element models, the cost of optimization quickly became prohibitive.

Even if the efficiency of the optimization algorithms could be improved to an acceptable level, there were good theoretical arguments to indicate that the cost of optimization was not constant and did not even increase linearly as the number of design variables increased, but instead increased at a quadratic or in some cases exponential rate.

If the structure was to be analyzed using the finite element method, it was commonly accepted that hundreds or even thousands of design variables would be desirable to provide complete design freedom. In other words, it was expected that one or more design variables would be associated with each finite element in the model.

By this time, enough computational experience had been documented to indicate that mathematical programming techniques applied to structural design were limited to perhaps 50 design variables.[18] In view of the need (or at least the desire) to design structures modeled with many more variables than this, it was clear that an impasse had been reached. It appeared that, although the generality of mathematical programming made this a most attractive design tool, practicality dictated that it was limited to problems defined by only a few design variables.

Growing Pains of the 1970's

In 1970, structural synthesis as defined and presented by Schmit was a decade old. There had been time to investigate many concepts within this framework and to begin to understand its limitations. The realization of these limitations was most graphically depicted in 1971 by Gellatly, Berke, and Gibson[19] when they referred to the '60's as a "period of triumph and tragedy" for structural optimization. In view of the growing disenchantment with the use of mathematical programming techniques, this was indeed an accurate and insightful statement!

Furthermore, an alternative approach was offered, and was presented in 1968 in analytical form by Prager and coworkers[20,21] and in numerical form by Venkayya, Khot, and Reddy.[22] This concept became popularly known as the "optimality criteria" approach.

The optimality criteria approach begins with the same general statement of the design problem; however, rather than working directly to minimize the objective function (weight, for example), one specifies a criterion such that if it is satisfied, subject to the constraints, then the design is defined as optimum. A common criterion is that the strain energy density in each member of the structure will be the same. Mathematically this leads to the problem of finding the design variables X such that

$$f(X_1) = f(X_2) = f(X_3) \ldots = f(X_n) = \text{const} \tag{6}$$

subject to:

$$g_j(X) \le 0 \qquad j = 1,m \tag{7}$$

$$X_i^l \le X_i \le X_i^u \qquad i = 1,n \tag{8}$$

where the equality constraints are omitted for brevity.

Now the Kuhn-Tucker necessary conditions for optimality[23] require that

$$\nabla F(X) + \sum_{j=1}^{m} \lambda_j \nabla g_j(X) = 0 \qquad j = 1,m \tag{9}$$

$$\lambda_j \ge 0 \qquad j = 1,m \tag{10}$$

$$\lambda_j g_j(X) = 0 \qquad j = 1,m \tag{11}$$

where ∇ is the gradient operator.

The Lagrange multiplier λ_j is positive only if constraint $g_j(X)$ is critical $[g_j(X) = 0]$. Therefore, using Eqs. (9) and (11), we can create a set of simultaneous nonlinear equations to solve for X and λ, subject to the nonnegativity requirements on λ_j.

The essence of the optimality criteria approach is first to establish the criterion that defines the optimum and then devise a recursive formula that leads, iteratively, to the desired solution. The problem is simplified somewhat if we can identify which constraints will be critical at the optimum because we know that these will have positive Lagrange multipliers. In practice, a priori knowledge of which constraints will be critical at the optimum is not essential, and these constraints may instead be identified during the iterative process.[24]

A special case is the design of statically determinate structures subject to stress limits only. Here it is known that each member will be fully stressed under at least one loading condition (or will be at its minimum gage), so for isotropic materials the uniform strain energy density assumption is clearly valid. Furthermore, it can be demonstrated that the fully stressed design of a statically determinate structure is also minimum-weight. Therefore, even though weight was not directly part of the optimality criterion, it was indirectly minimized. In the more general case, the problem of identifying which constraint would be critical at the optimum presented, at least theoretically, a limitation to this concept.

Although the optimality criteria approach was largely intuitive, it was shown to be quite effective as a design tool. Its principal attraction was that the method was easily programmed for the computer, was relatively independent of problem size, and usually provided a near-optimum design with as few as 15 detailed structural analyses. This last feature represented a remarkable improvement over the number of analyses required for mathematical programming methods to reach a solution.

Because of this vast improvement in efficiency, considerable research effort was devoted to optimality criteria concepts in the early and mid-1970's. An excellent description of the concepts is provided by Kiusalaas.[25] The works of Prager,[26] Venkayya et al.,[27] Berke and Khot,[28] and Isakson et al.[29] are indicative of the capabilities of that time.

The two competing design concepts of this period provided a choice that was less than desirable since neither concept was clearly superior for application to problems of practical interest. Mathematical programming offered an extremely general tool. Also, it was attractive from a theoretical viewpoint in that no assumptions were required about the nature of the optimum. One could simply approach the design process as a general nonlinear-constraint-minimization problem and let the "optimizer" lead where it would. On the other hand, optimality criteria had no clear theoretical basis. It was known that these techniques would, on occasion, lead to nonoptimum designs and would even diverge from the solution. However, this behavior was usually associated with specially constructed test problems and appeared to be an academic more than a practical concern. Most important, optimality criteria offered a solution for more practical design problems, a feature that often overshadowed the limitations of the method.

The strengths of the two methods suggested a natural separation of the design problem, where optimality criteria would deal with a large number of system variables and mathematical programming would solve the component-design problem. This approach was pursued with success by Sobieszczanski (Sobieski) and Leondorf[30] in the design of fuselage structures. This work is important, not only because it combined both methods, but also because it seemed to build a common ground where researchers in both areas began to look more closely for a fundamental relationship between the methods. This is not to suggest that there was no cross flow of information before this. Indeed, about 1970, Berke* suggested to this author the desirability of looking for a theoretical link between the two approaches. The motivation for such an effort would be to understand when optimality criteria would and would not provide a theoretical optimum. Similarly, in that same time period, Schmit† was encouraging his graduate students, including this author, to investigate "approximation concepts" as a mechanism to improve the efficiency of using mathematical programming techniques.

In 1973 Schmit and Farshi presented a concise statement of the approximation concept approach to structural synthesis using mathematical programming.[31] Several concepts are presented in Ref. 31, but perhaps the most far-reaching is the use of intermediate variables to provide high-quality, explicit approximations to the original problem. The principal technique was use of the reciprocal of the member sizes as design variables; for example, given a bar element, the elongation $\delta = PL/AE$ is inversely proportional to the cross-sectional area. The gradient of the elongation with respect to the design variable A is

$$\mathrm{d}\delta/\mathrm{d}A = -PL/A^2E \tag{12}$$

Replace the design variable A by its reciprocal $B = 1/A$. Now

$$\delta = PLB/E \tag{13}$$

and

$$\mathrm{d}\delta/\mathrm{d}B = PL/E = \mathrm{const} \tag{14}$$

Because displacement (or stress, which is linearly related to displacement) is a constraint on the design, this simple change of variables has converted a nonlinear constraint to a linear constraint. In general, when this bar is a part of an indeterminate structure, there is coupling between the design variables, and the reciprocal formulation is approximate, being precise only in the case of statically determinate structures.

Note that, if the objective to be minimized is weight, $W = \rho AL$, the change of variables leads to a nonlinear, but still explicit, objective $W = \rho L/B$. Therefore, an approximation is created which has a nonlinear objective function with linear constraints, all of which are explicit functions of the intermediate variables B and are easily and economically evaluated.

The approximation to the original problem having been created, the approximating functions are used in the optimization. Once the optimum solution to the approximate problem is found, a precise finite element analysis is performed and a new approximation is created. In this fashion, the final optimum is obtained iteratively. Thus a technique is provided in which all the features of the original problem are retained in such a way that a sequence of approximate optimizations leads to a precise solution. Because the approximate problem requires little effort for function and gradient evaluation, mathematical programming techniques can be used for this subproblem.

A detailed description of approximation concepts together with numerous examples was presented by Schmit and Miura in 1976.[32] The techniques were applied to structures made of bar and membrane elements and subjected to strength and displacement constraints under multiple loading conditions. Using these methods, the optimum design of a statically determinate structure is obtained with only one detailed analysis. For the design of indeterminate structures, the number of detailed analyses reported in Ref. 32 is typically 10. However, the reported results, viewed in the context of practical applications, show that a very near optimum design (optimum within machining tolerances) is obtained in as few as 3 detailed analyses, even for highly redundant structures. Furthermore, this efficiency appears to be independent of problem size. This was a major development that allows the designer to retain the generality of mathematical programming while solving problems of practical size.

During the late 1970's, development continued in both optimality criteria and mathematical programming approaches to structural optimization. In terms of understanding the automated design process, perhaps the most significant work was in the area of reconciling the mathematics of the two basic concepts. The definitive work of Fleury[33-35] and Fleury and Sander[36] offers fundamental insight into the mathematical basis of both approaches and, in fact, shows a common basis in the duality of the original problem statement. This work principally shows that optimality criteria are valid for a mathematically separable problem and, as such, may be viewed as a special case of mathematical programming.

Indeed, it appears that the combination of approximation concepts and dual methods[37-39] has provided the "best of both worlds" for a large class of design problems.

The Present

The discussion thus far has dealt with development of the state-of-the-art in structural optimization using numerical techniques. Particularly significant contributions have included the initial nonlinear programming approach to structural synthesis, development of optimality criteria, approximation concepts, and, most recently, the coalescence of ideas through the combining of approximation concepts and dual methods.

Although there has been major progress, much remains to be done. The purpose of this section is twofold. The first is to briefly identify the state-of-the-art as it relates to the disciplines incorporated in the synthesis. These include the optimization techniques, an analysis procedure (typically the finite element method), and the mechanism through which these disciplines are combined. The second purpose here is to identify the state-of-the-art as it applies to practical applications and the potential for future applications of these techniques to structural design.

Mathematical programming (where optimality criteria are now considered as a special case) has matured significantly in the last 20 years even though most of the basic algorithms were known in some form in the early 1960's. Although these algorithms were developed primarily by the operations research community, major modifications have often been made to provide an efficient and reliable tool for structural design. There are two reasons for these modifications.

First, there is a subtle but fundamental difference between developing a mathematical algorithm, together with proofs of convergence indicating its efficiency, and actually making this algorithm usable for engineering design on a digital computer. An excellent recent example of the practicalities of using mathematical programming in design is found in Khachian's algorithm for solving linear programming problems.[40] This algorithm was highly publicized in the press as a method that would converge for the worst-case problem at a polynomial rate, as compared with Dantzig's time-honored Simplex method,[41] which had a much less desirable exponential convergence rate. Yet the solution to the trivial problem of maximizing the single variable X subject to the limit that X not exceed unity requires 1 iteration using the Simplex method and 82 iterations using Khachian's method.[42] Furthermore, with Khachian's algorithm, many significant figures must be retained in the computer for the solution of problems of even

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modest size. The message here is that the theoretically more attractive method may not be best in practice. In engineering design, it is often necessary to take liberties with the mathematical properties of the problem in order to devise a workable algorithm. For example, although an inequality constraint $g(X) \leq 0$ is mathematically "active" only when $g(X)$ is precisely zero, in an engineering sense the value $g(X) = -0.05$ may be considered critical. This is because material properties, failure criteria, and loading environments are virtually never known with greater precision. Consequently, from an engineering viewpoint, a mathematically precise optimum is seldom meaningful.

The second reason is that the cost of one analysis of a proposed design can be very high; therefore, the rate of convergence of the optimization algorithm is important. A method that converges quickly to a near optimum is usually preferred to one that converges slowly to a precise optimum. This argument is somewhat less important when approximation concepts are used because the problem solved by the optimization code is computationally efficient. As will be seen in the following paragraphs, however, approximation concepts do not apply to all structural optimization problems, and so there remains a need for optimization algorithms with rapid-convergence characteristics.

The state-of-the-art in mathematical programming is such that the optimization algorithm is usually not a critical issue so long as a modern algorithm is used. For example, the steepest-descent method should never be used for unconstrained minimization because very little additional effort is required for the use of conjugate direction methods, with their improved convergence characteristics. A general overview of mathematical programming methods is presented in Ref. 43. References 44-48 present comparative studies of nonlinear programming codes and attempt to rank several available programs. In reviewing this literature, a word of caution is in order. The problems solved in these studies are often quite trivial in terms of computer time for one analysis. Also, they often bear no relationship to engineering design, and those studies that do include "engineering" problems consider examples that do not require significant resources for analysis. The figure of merit used to rank the codes is usually a combination of CPU time, preparation time, ease of use, etc., in addition to the number of function evaluations required. Consequently a program may appear quite good while requiring hundreds of function evaluations to solve a problem of only three or four variables. If, however, one function evaluation consists of solving a complex analysis problem, for example, flutter or postbuckling analysis, the cost of optimization using such a program would be prohibitive. This suggests that, for practical design, only two criteria are meaningful. First, does the program reliably achieve a near-optimum design (as noted earlier, a precise optimum is seldom meaningful) and second, does the program use few enough function (and gradient) evaluations to be economically usable for the design task at hand? The manner in which the optimization and analysis programs are coupled together to create the design program will determine the ultimate utility of these techniques, and the engineer's ability to match the optimization algorithm to the design problem usually has the most important impact on the overall result.

Most large computer installations maintain one or more nonlinear programming codes. The designer should begin by experimenting with these, remembering an additional word of caution. The same algorithm programmed by two different people will often differ in reliability and computational efficiency by more than an order of magnitude. This is because we are attempting to numerically model on the computer much of the judgment, experience, and intuition of a good design engineer. A numerical optimization code, although based on a particular algorithm, will reflect the ability of the person who programmed the code to model this highly complex decision process. If the locally available mathematical programming codes are not satisfactory, Refs. 12 and 44-49 identify numerous additional sources. Any cost to obtain a particular code is usually nominal compared with the cost of developing a new optimization code.

To summarize, the state-of-the-art in mathematical programming is such that engineers should no longer find it necessary to develop their own programs. It should be expected, however, that with experience, they may wish to tailor an existing code to solve their particular design problem more efficiently.

The second essential ingredient to structural synthesis is the analysis capability. It is assumed that the engineer is familiar with and will use a finite element program in design, although for specific cases other analysis techniques may be employed. The state-of-the-art in finite element analysis is becoming quite advanced, probably the most widely used program being NASTRAN.[50] Here, however, there is a missing ingredient, namely, the ability to obtain gradient (sensitivity) information as part of the analysis, even though the technology for calculating this information is well in hand.[31,51,52] Although sensitivity information was obtained using NASTRAN as early as 1974,[53] obtaining this information is not an integral part of the program, with the efficiency that would result if it were. Several other programs exist that do provide gradient information and in most cases also include optimization in an integrated analysis/design code. These include, as examples, a general programming framework,[54] and programs ACCESS,[55] ASOP,[56] EAL,[57] PARS,[58] SAVES,[59] SPAR,[60] STARS,[61] and TSO.[62] Although providing a remarkable design capability, these programs have not gained wide acceptance in the design community at this time.

The final and most critical ingredient to structural synthesis is the mechanism by which the analysis and optimization programs communicate. The most direct approach is to simply couple the optimization code to the analysis code and treat the problem in the form originally presented by the engineer. Any gradient information would be calculated by finite difference, and each function evaluation required by the optimization program would be a completely new analysis of the structure. This "black box" approach is actually quite reasonable for a large percentage of design tasks. If one is designing a structural component, and the analysis is not expensive but several design variables are used and multiple loading and constraint conditions are imposed, this direct approach is particularly useful.

For the design of more complex structural systems, the cost of repeated analyses usually precludes the black box approach. Here the first step is to incorporate gradient computations into the finite-element-analysis code. It is conceptually straightforward to provide this information for stress, displacement, frequency,[63,64] and flutter[65] constraints. Having done this, there remains considerable effort to develop an efficient and reliable design program. It is here that the approximation concepts of Refs. 31 and 32 play a fundamental role. By generating the approximate problem, one drastically reduces the computational resources needed to reach the solution. Furthermore, through the use of dual methods[37-39] it is now possible to treat discrete variables in the automated design process, allowing as design variables the number of plies in a composite laminate or the selection of panels from available gage sizes. These recent developments are not universally applicable, however, and it is important to understand their limitations.

First consider the approximation concepts. The quality of the approximation is directly dependent on the analysis model or, more precisely, on the choice of intermediate design variables. For example, in designing structures made of bar or membrane elements, the basic design variables are the cross-sectional area A and the member thickness t. By picking the intermediate design variable $B = 1/A$ or $B = 1/t$, the first-order Taylor series approximation to the displacement with respect to this variable is precise for statically determinate structures and is a good approximation for indeterminate structures. This is because the element stiffness matrix is the product of the original scalar variable, A or t, and an invariant matrix that depends only on the material properties and geometry of the structure. Similarly, if the structure is composed of plate bending elements, a good intermediate variable would be $B = 1/t^3$.

Now consider the design of a rectangular frame element, where the design variables are the width and height of the element. Because the element stiffness matrix contains the moments of inertia, the design variables are interdependent even at the element level. This suggests that the first-order Taylor series expansion will not be as accurate a representation of the structural response as before, and consequently the applicability of approximation concepts begins to break down. This difficulty is compounded with geometric design variables, whether they be joint locations for truss structures or the shape of a solid, modeled by three-dimensional isoparametric elements.

This same difficulty is associated with the use of dual methods as an optimization technique. These methods theoretically require that the design problem be mathematically separable. Separability requires that $F(X) = f_1(X_1) + f_2(X_2) + ... f_n(X_n)$, where $f_i(X_i)$ may be a complex function but does not depend on X_j, $j \neq i$. In the case of bar and membrane elements, this relation exists with respect to the objective function and is approximated for the constraints by virtue of the first-order approximation.

The limitations of these newest techniques preclude their direct application to many design situations, but it is sometimes possible to utilize them in a multilevel optimization approach. In Refs. 66 and 67, the approximation concepts are used for system-level variables, whereas component-level optimization is performed as a direct optimization subproblem. In another form of the multilevel concept this approach is reversed. Reference 68 offers an approach to configuration optimization in which the nodal coordinates are treated as system design variables using a direct optimization approach. For each proposed configuration, the member sizes are updated as a subproblem using approximation techniques. The fundamental issue, however, remains to be resolved. That is, for a given design problem in which each member is described by more than one design variable, or where geometric variables are considered, what is the best choice of intermediate variables to provide a high-quality approximation to the problem?

In summary, the state-of-the-art provides a remarkable capability for automated structural synthesis. The technology now exists to efficiently design structures defined by several hundred design variables under multiple loading conditions and subject to sizing, stress, displacement, stability, frequency, and flutter constraints. References 69-80, reviewing the recent developments in this field, attest to the maturing state-of-the-art in this discipline. Indeed, as stated by Schmit in Ref. 81, structural synthesis has matured from an "abstract concept to [a] practical tool."

The degree to which structural synthesis has become a practical tool is apparent in such capabilities as the PASCO program for composite-panel design.[82] This program is capable of designing a wide variety of stiffened panels under multiple loading conditions, and the validity of the results is supported by an extensive experimental study.[83] An example optimization of a problem of significant size and complexity is found in Ref. 84, which reports the preliminary design of an advanced transport aircraft, including strength, stiffness, and aeroelastic constraints. Both these capabilities were developed or supported by NASA. Despite the demonstrated success of this technology, it remains difficult to identify industrial organizations that utilize formal optimization techniques to a significant and continuing degree. It must be a source of frustration to developers of this technology that millions of dollars is currently being invested in computer graphics and computer-aided design, while the opportunity to fully automate major portions of the design process is being virtually ignored. In his Wright Brothers lecture,[85] Ashley is able to identify numerous applications of optimization to aerospace problems and references several design studies. Despite difficulty in identifying concrete examples of the use of optimization in the design of aircraft that have actually been built, he makes a compelling argument for the use of formal optimization techniques and compiles enough evidence to conclude by stating that this technology offers a "cosmic opportunity" for the future.

The Future

Any attempt to predict the future in such a dynamic discipline can only be futile. It is perhaps sufficient to be able to claim with some certainty that there *is* a future for structural synthesis. This has resulted from a simultaneous maturing of three distinct parts, numerical optimization, finite element analysis, and the general concepts underlying design synthesis. In addition, the need for lightweight, economical structures is greater than ever before, now principally motivated by energy and finite resource considerations.

Although it is unreasonable to predict the precise form of the structural synthesis discipline of the future, it is possible to identify some needs, recognizing that this prediction, too, is speculative and incomplete. To this end, the three components of structural synthesis will be addressed individually.

In the mathematical programming discipline, two clear needs must be addressed. First is the need for public availability of well written and documented computer codes incorporating a variety of today's state-of-the-art algorithms. Whether by government edict, by professional society guideline, or by simple evolution, a clear set of guidelines needs to be accepted by developers of mathematical programming software for the coding, testing, and documenting of this capability. These guidelines will go far to eliminate the frustration experienced by practitioners who obtain mathematical programming codes, at some effort and expense, only to find them totally unsatisfactory for practical engineering design.

The second obvious need is for development of algorithms that are efficient for the solution of large-scale nonlinear programming problems. This will alleviate to a degree the need for high-quality approximations in structural synthesis. Recent literature in mathematical programming suggests that significant progress can be expected in the future.[86]

In the area of finite element analysis, it has already been pointed out that linear elastic analysis is well developed in such programs as NASTRAN, but that gradient information is often unavailable. Providing gradient information as part of the analysis must be the first priority if structural synthesis is to become widely used by practicing engineers. This is, however, primarily an economic issue rather than a technology question, and the capability will undoubtedly be provided in one or more of the large-scale finite element codes when the market demands it strongly enough or when far-sighted code developers emerge who are willing to risk the necessary development of this advanced design capability. As regards development of methodology, analysis of composite materials (particularly failure criteria) remains an important issue. Both time-dependent and large-deformation nonlinear analysis, for use within the structural synthesis framework, may well become another important research area. As another example, the questions of stochastic loading and time-parametric constraints need to be addressed within the structural analysis/synthesis context far more than in the past. Finally, efficient reanalysis techniques need to be pursued, particularly as related to damaged structures, and these aspects need to be incorporated into the design process.

The means by which mathematical programming and analysis codes are integrated into a structural synthesis capability will surely continue to be an area of intense research and development. Several areas are identifiable where significant progress may be expected. First is a means of dealing effectively with such problems as frame elements defined by more than one design variable, either directly through the proper choice of intermediate variables, or in a multilevel design approach. Progress here may lead to an effective means of dealing with configuration variables, an area of immense payoff because of the potentially large design improvements possible. The combinatorial, or topological, problem is another intriguing research area, where, in addition to treating member sizes and nodal coordinates in the structure as variable, one determines the actual element-node connectivity. Another area of fruitful research is the design of structures from a reliability viewpoint. Considerable work has already been reported,[87] yet much remains to be done. This is an especially interesting subject in the sense that it may offer fundamental insight into the stochastic loading and parametric constraint problems. Finally, the need to design structures for survivability when damaged is only beginning to be addressed.[88] In view of increasing emphasis on survivability, this topic deserves high priority.

This short list of possible future developments in structural synthesis is by no means complete, but it does indicate the phenomenal amount of effort that lies ahead. Contrary to the sometimes expressed view that the computer will eliminate engineering jobs, it appears that there is ample work for the future!

As pointed out in the introduction, the concept of structural synthesis using mathematical programming offered a new design philosophy that only today is beginning to be broadly applied. The works referenced here cannot do justice to the many researchers who have contributed to this technology (see Refs. 69-80 for an extensive list of works in the 1970-1980 time frame). In addition to applications in structures, this work has contributed to advancement in other engineering design. References 12 and 89-91 offer an indication of the breadth of applications that has resulted, to a very large degree, from the leadership of researchers in structural synthesis. Indeed, Refs. 92 and 93 are examples of an almost direct application, to an unrelated design discipline, of concepts presented in Refs. 31 and 32. This expansion may be expected to continue and grow, and as researchers from various disciplines discover common ground, the ultimate design goal of integrated system synthesis may begin to evolve.

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